BUSINESS MATHEMATICS & STATISTICAL INFERENCE – SEMESTER-2

Marks

Q. 2 (a) (i) $\sqrt[6]{64x^{18}y^{12}}$

$= (64x^{18}\, y^{12})^{1/6}$ 0.5

$= (64)^{1/6}\, x^{18/6}\, y^{12/6}$ 0.5

$= 2x^3\, y^2$ 1.0

(ii) $\dfrac{(x+3)^2}{(x+1)^2} \div \dfrac{x^2-9}{x^2-1}$

$\dfrac{(x+3)^2}{(x+1)^2} \times \dfrac{(x^2-1)}{(x^2-9)}$ 0.5

Now $(x^2-1) = (x+1)(x-1)$ 1.0

And $(x^2-9) = (x+3)(x-3)$ 1.0

Therefore:

$\dfrac{(x+3)^2}{(x+1)^2} \times \dfrac{(x+1)(x-1)}{(x+3)(x-3)}$ 0.5

$\dfrac{(x+3)(x-1)}{(x-3)(x+1)}$ 1.0

(b) $2x^2 - 3x - 2 = 0$

$2x^2 - 4x + x - 2 = 0$ 1.0

$2x(x-2) + 1(x-2) = 0$ 0.5

$(x-2)(2x+1) = 0$ 0.5

If $x - 2 = 0$ then $x = 2$ Ans 1.0

If $2x + 1 = 0$ then $x = -1/2$ Ans 1.0

(c) P= Rs.100,000 , i= 0.18 p.a. = 0.045 per qtr , n = ? A= 250,000 1.0

$A=P(1+i)^n$ 1.0

$250{,}000=100{,}000(1+0.045)^{4n}$

$2.5=(1.045)^{4n}$ 1.0

Taking log on both sides:

$4n \log 1.045 = \log 2.5$

$4n \times 0.019116 = 0.39794$ 1.0

= 5.2 years Ans 1.0

Marks

Q.3 (a)

$x^2 - 3x + 2 \geq 0$

$x^2 - x - 2x + 2 \geq 0$

$x(x-1) - 2(x-1) \geq 0$ — 1.0

$(x-1)(x-2) \geq 0$ — 1.0

$(x-1)(x-2) = 0$ then $x = 1$ or $x = 2$ — 1.0

$(x-1)(x-2) > 0$ then $x > 1$ and $x > 2$ — 0.5

$(x-1)(x-2) > 0$ then $x < 1$ and $x < 2$ — 0.5

$x \leq 1$ and $x \geq 2$ Ans. — 1.0

(b) (i) Revenue = $R = f(p) = pq$ — 0.5

$= p(10,000 - 125p)$

$= 10,000p - 125p^2$ — 1.0

Step 1:

$R' = 10,000 - 250p$ — 0.5

Step 2:

Taking $R' = 0$, we get: — 0.5

$10,000 - 250p = 0$

$p = 40$ — 1.0

Step 3:

$R'' = -250$ — 0.5

Step 4:

Since $R''(p=40) = -250 < 0$ — 0.5

Hence the revenue of the firm is Maximized when $p = 40$ — 0.5

(ii) $R(max) = f(40) = 10,000(40) - 125(40)^2$ — 1.0

$= 400,000 - 200,000 = 200,000$ — 1.0

(iii) $q = 10,000 - 125 \times 40$ — 1.0

$= 10,000 - 5,000 = 5,000$ units — 1.0

Marks

(c) R = Rs. 15,000; n = 20 years = 240 months;

i = 18 % p.a. =0.015 p.m. ; A =? 1.0

$$A = R\frac{\{(1+i)^n-1\}}{i(1+i)^n}$$ 1.0

$$A = \frac{15{,}000 \times \{1+0.015)^{240}-1\}}{0.015(1+0.015)^{240}}$$

$$A = \frac{15{,}000 \times \{35.6328-1\}}{0.015 \times 35.6328}$$ 2.0

$$A = \frac{15{,}000 \times 34.6328}{0.015 \times 35.6328}$$

= 971936 2.0

Alternatively :

A= 15,000 (64.795732) using relevant table

= 971,935.98Ans

Q.4 (a) $\text{Mode} = l + \frac{h(f_m - f_1)}{2f_m - f_1 - f_2}$ 0.5

From given table: $l = 30.5$, $h = 5$, $f_m = 20$

$f_1 = 13$ and $f_2 = 18$ 0.5 mark each = **2.5** (0.5 x 5)

$$= 30.5 + \frac{5x(20 - 13)}{2x20-13-18}$$

= 30.5 + 35/9 1.0

= 30.5 + 3.89

=34.389 Ans. 1.0

(b) 19, 25, 10, 27, 16

Mean = (10+16+19+25+27)/5 =97/5 = 19.4 1.0

10, 16, 19, 25, 27 (arranging above data in ascending order) therefore:

Median = 19 1.0

$$\text{Variance} = \frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2$$ 0.5

= (100+256+361+625+729)/5 -19.4^2 = 414.2 -376.36 0.5

= 37.84 1.0

Marks

Standard Deviation = $\sqrt{37.84}$ =6.1514 0.5

Pearson's Coefficient of Skewness = 3(Mean – Median)/SD 0.5

=3(19.4 – 19)/6.1514 0.5

=0.1951 Ans. 0.5

(c)

Marks	No. Of Students (fi)	x	fx	fx^2
1 – 20	12	10.5	126	1,323
21 – 40	19	30.5	579.5	17,674.75
41 – 60	22	50.5	1,111	56,105.5
61 - 80	27	70.5	1,903.5	134,196.75
81 - 100	20	90.5	1,810	163,805
	100		**5,530**	**373,105**
Marking Plan ⟹		**0.5**	**+1**	**+1.5**

3.0

Mean = μ = ∑fx/∑f = 5530 /100 = 55.3 1.0

Population Variance = $\sum fx^2/\sum f - \mu^2$ 0.5

= 373105/100 – 55.3 ^{2}

= 3731.05 –3058.09 0.5

= 672.96 1.0

Standard deviation = $\sqrt{672.96}$ = 25.94 0.5

Coefficient of variance = (SD / Mean) x 100

= (25.94 / 55.3) x 100 0.5

= 46.9 % Ans 1.0

Q.5 (a)

x	Y	xy	X^2	Y^2
85	92	7820	7225	8464
92	80	7360	8464	6400
63	50	3150	3969	2500
45	50	2250	2025	2500
88	85	7480	7744	7225
56	52	2912	3136	2704
429	409	30972	32563	29793
Marking Plan ⟹		**+1.0**	**+1.0**	**+1.0**

3.0

Marks

Coefficient of Correlation $= \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$ **1.0**

$= \frac{6 \times 30972 - 429 \times 409}{\sqrt{(6 \times 32563 - 429^2)(6 \times 29793 - 409^2)}}$ **1.0**

= 0.909 **1.0**

Positive (direct) and strong relationship exits between x and y. **1.0**

(b)

Months	Annual Sales (Units)	3 Monthly moving averages	
January	450		
February	440	450	0.5
March	460	437	0.5
April	410	417	0.5
May	380	397	0.5
June	400	383	0.5
July	370	377	0.5
August	360	380	0.5
September	410	407	0.5
October	450	443	0.5
November	470	470	0.5
December	490		

(c)

Total Possible Outcomes = 26x25x24x9x8x7 1.0

= 7,862,400 1.0

Favourable outcomes = 5x25x24x8x7x5 1.0

= 840,000 1.0

Probability = Favorable outcomes/ Total Possible Outcomes 0.5

=7,862,400/840,000 0.5

= 0.1068 Ans 1.0

Marks

Q. 6 (a)

μ = 65, σ^2 = 60, Top 10% = A Graders, next 20% = B Graders, 1.0

Area = 30% above the curve 0.5

Area = 70% bellow the curve 0.5

Z = 0.52 1.0

$X = \mu + z\sigma$ 1.0

= 65 +0.52 x 7.7453 = 65 + 4.02 = 69.027 ≅ 70 Ans 1.0

(b)

1. Ho : μ = 4 1.0
2. H_1 : μ > 4 1.0
3. α = 0.05
4. Critical Region:

   t > tα , v = degree of freedom = n-1 = 12 – 1 = 11 0.5

   t > 1.796 0.5
5. Computations:

   n = 12, s = 1.5 mg , Sample Mean (x)= 4.3 mg

$$t = \frac{(\bar{x} - \mu)}{s/\sqrt{n}}$$

$$t = \frac{(4.3 - 4)}{1.5/\sqrt{12}}$$ 1.0

= 0.69 1.0

6. Decision:

Computed value did not fall in the critical region so we accept Null Hypothesis. 1.0

Marks

(c) N= 4, n= 2
Maximum no. of samples = N^n
= 4^2 = 16 — 1.0

S.No.	Samples	Sample Mean	
1	(4, 4)	4	0.25
2	(4, 6)	5	0.25
3	(4, 8)	6	0.25
4	(4,10)	7	0.25
5	(6, 4)	5	0.25
6	(6, 6)	6	0.25
7	(6, 8)	7	0.25
8	(6,10)	8	0.25
9	(8, 4)	6	0.25
10	(8, 6)	7	0.25
11	(8, 8)	8	0.25
12	(8,10)	9	0.25
13	(10, 4)	7	0.25
14	(10, 6)	8	0.25
15	(10, 8)	9	0.25
16	(10, 10)	10	0.25

Sampling Distribution of Mean: — 2.0

Sample Mean (x)	4	5	6	7	8	9	10
f(x)	1/16	2/16	3/16	4/16	3/16	2/16	1/16

THE END